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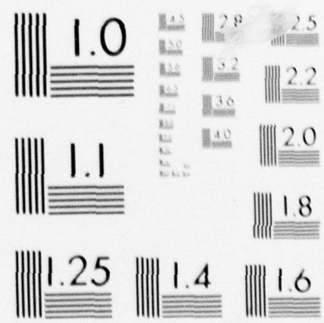

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EFFICIENT ESTIMATION OF MULTIVARIATE MOVING AVERAGE AUTOCOVARIANCES

By H. J. Newton
Texas A&M University

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This paper proposes a method for estimating the autocovariances of a d-dimensional moving average process of order q. The estimators have the same asymptotic covariance matrix as those obtained by maximizing a Gaussian likelihood, and are obtained by performing a generalized least squares regression of the periodogram on the autocovariance, thus extending Parzen's (1971) estimators for d = 1.

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Efficient Estimation of Multivariate Moving

Average Autocovariances

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Summary

This paper proposes a method for estimating the autocovariances of a d-dimensional moving average process of order q. The estimators have the same asymptotic covariance matrix as those obtained by maximizing a Gaussian likelihood, and are obtained by performing a generalized least squares regression of the periodogram on the autocovariances, thus extending Parzen's (1971) estimators for d = 1.

Some key words: Moving average model; multiple time series; autocovariance, generalized least squares; spectral density

1. INTRODUCTION

Let $\{\varepsilon(t), t\in\mathbb{Z}\}$ be a d-dimensional white noise series on the integers with mean zero and positive definite covariance matrix Σ. A d-dimensional moving average process $\{Y(t), t\in\mathbb{Z}\}$ of order q is defined by

$$Y(t) = \sum_{k=0}^{q} B(k)\,\varepsilon(t-k) ,$$

where the B(k)'s are d x d real matrices, and $B(0) = I_d$, the d-dimensional identity matrix.

Phadke and Kedem (1978) survey methods for obtaining estimators of Σ and the B(k)'s from a sample realization Y(1), ..., Y(T) . The methods generally consist of maximizing an exact or approximate Gaussian likelihood to obtain estimators $\hat{B}(1), \ldots, \hat{B}(q)$, and Σ that are called least squares estimators or a maximum likelihood identification.

Parzen (1971) proposed a method for estimating the moving average autocovariances $R(v) = E(Y(t)Y^T(t+v))$ for d = 1 by capitalizing on the linear relationship between the R(v)'s and the spectral density $f(\cdot)$ of Y(t) :

$$f(\omega) = \frac{1}{2\pi} \sum_{|v|\leq q} R(v)e^{-iv\omega} , \quad \omega\in[-\pi,\pi] , \tag{1}$$

where

$$R(v) = R^T(-v) = \begin{cases} \sum_{k=0}^{q-v} B(k)\,\Sigma\, B^T(k+v) , & v = 0, \ldots, q \\ 0 , & v > q . \end{cases}$$

Newton, in a Buffalo PHD thesis showed that the Parzen estimators have the same asymptotic covariance matrix as those obtained by maximumizing the approximate Gaussian likelihood.

The purpose of this paper is to provide an extension of the Parzen method for d > 1 . The extension is given in section 2 while conclusions are given in section 3.

2. GENERALIZED LEAST SQUARES ESTIMATORS OF R(·)

Consider the sample spectral density or periodogram

$$f_T(\omega_j) = \frac{1}{2\pi T} \sum_{s,t=1}^{T} Y(s)Y^T(t)\, e^{i(s-t)\omega_j}, \quad \omega_j = \frac{2\pi j}{T}, \; j=1, \ldots, \left[\frac{T-1}{2}\right] = N,$$

where $[x]$ denotes the greatest integer less than or equal to x. By inspection of Hannan's Corollary 1 (see Hannan (1970), p. 249)

$$\lim_{T\to\infty} E(\mathrm{vec}(f_T(\omega_j))) = \mathrm{vec}(f(\omega_j)) \tag{2}$$

$$\lim_{T\to\infty} \mathrm{Cov}(\mathrm{vec}\,(f_T(\omega_j)),\ \mathrm{vec}\,(f_T(\omega_k))) = \delta_{jk}\, f^T(\omega_j) \otimes f(\omega_j),$$

where δ is the Kronecker delta, $C \otimes D$ is the Kronecker product $(C_{jk}D)$ of the matrices C and D, and if A is an $n \times m$ matrix with columns $a_1, \ldots, a_m$, then vec (A) is the $nm \times 1$ vector $(a_1^T, \ldots, a_m^T)^T$.

Thus combining (1) and (2) we can write

$$\mathrm{vec}(f_T(\omega_j)) = \mathrm{vec}\,\Big(\sum_{|v|\le q} R(v)\, e^{-iv\omega_j}\Big) + \varepsilon_T(\omega_j), \quad j = 1, \ldots, N, \tag{3}$$

where $\varepsilon_T(\omega_1), \ldots, \varepsilon_T(\omega_N)$ are asymptotically uncorrelated d^2dimensional random variables with asymptotic mean zero and variance $f^T(\omega_j) \otimes f(\omega_j)$.

To write (3) in a regression form we define the permutation matrices P and G as follows. Let e_j denote a vector of zeros with a 1 in column j. The k^{th} row of the $d^2 \times d^2$ matrix P is given by $e^T_{jd+\ell}$ where $\ell = [\frac{k-1}{d}]$ and $j = (k-1) - d\,[\frac{k-1}{d}] \equiv \mathrm{mod}\,(k-1, d)$. Letting $s = d(d+1)/2$, the k^{th} row of the $d^2 \times s$ matrix G is e_i^T where $i = s - (d-m+1)(d-m+2)/2 - (\ell-m-1)$, with $\ell = \max\,(u,v)$, $m = \min\,(u,v)$, and $u = \mathrm{mod}\,(k-1,d) + 1$, $v = [\frac{k-1}{d}] + 1$.

Then

$$\mathrm{vec}\,(R(-v)) = \mathrm{vec}\,(R^T(v)) = P\,\mathrm{vec}\,(R(v)), \; v=1, \ldots, q$$
$$\mathrm{vec}\,(R(0)) = G\,\mathrm{vech}\,(R(0)), \tag{4}$$

where vech (R(0)) contains the distinct elements of the symmetric matrix R(0) and is obtained by applying the vec operator to only the lower triangular portion of R(0).

Using (4) we write

$$\mathrm{vec}\,\Big(\sum_{|v|\le q} R(v)e^{-iv\omega_j}\Big) = G\,\mathrm{vech}\,(R(0)) + \sum_{v=1}^{q} (Pe^{iv\omega_j} + I_{d^2}e^{-iv\omega_j})\,\mathrm{vec}\,(R(v))$$
$$= X(\omega_j)\,R, \; j = 1, \ldots, N, \tag{5}$$

where $R^T = ([\mathrm{vech}\,(R(0))]^T, [\mathrm{vec}\,(R(1))]^T, \ldots, [\mathrm{vec}\,(R(q))]^T)$ contains the distinct elements of R(v), $|v|\le q$, and

$$X(\omega_j) = (G,\ Pe^{i\omega_j} + I_{d^2}e^{-i\omega_j}, \ldots, Pe^{iq\omega_j} + I_{d^2}e^{-iq\omega_j})$$

is a $d^2 \times (qd^2+s)$ matrix of known complex numbers.

Writing (3) for $j = 1, \ldots, N$ and using (5), we have

$$f = XR + \varepsilon$$

where $f = ([\mathrm{vec}\,(f_T(\omega_1))]^T, \ldots, [\mathrm{vec}\,(f_T(\omega_N))]^T)^T$ is an $Nd^2 \times 1$ observation vector, $X = (X^T(\omega_1), \ldots, X^T(\omega_N))^T$ is an $Nd^2 \times (qd^2+s)$ design matrix, and $\varepsilon = (\varepsilon_T^T(\omega_1), \ldots, \varepsilon_T^T(\omega_N))^T$ is an $Nd^2 \times 1$ error vector with block diagonal asymptotic covariance matrix $V = \mathrm{DIAG}\,(V(\omega_1), \ldots, V(\omega_N))$, where $V(\omega_k) = f^T(\omega_k) \otimes f(\omega_k)$.

A consistent estimator of $V(\omega_k)$ is given by

$$\hat{V}(\omega_k) = \hat{f}^T(\omega_k) \otimes \hat{f}(\omega_k)\ , \text{ where } \hat{f}(\omega_k) = \frac{1}{2\pi} \sum_{|v| \leq q} R_T(v) e^{-iv\omega_k}\ ,$$

and

$$R_T(v) = R_T^T(-v) = \frac{1}{T} \sum_{t=1}^{T-v} Y(t) Y^T(t+v)\ ,\ v = 0, \ldots, T-1\ .$$

Then the generalized least squares estimator $\hat{R}$ of R is the solution to $(X^* \hat{V}^{-1} X) R = X^* \hat{V}^{-1} f$,

or

$$\hat{R} = \left[\sum_{t=1}^{N} X^*(\omega_t) \hat{V}^{-1}(\omega_t) X(\omega_t) \right]^{-1} \sum_{t=1}^{N} X^*(\omega_t) \hat{V}^{-1}(\omega_t) \operatorname{vec}(f_T(\omega_t))\ ,$$

where A^* denotes the complex conjugate transpose of the complex matrix A.

Using arguments similar to the case d = 1 and results for information matrices of multiple time series given by Newton (1978), one can show that $\sqrt{T}\,(\hat{R}-R)$ is asymptotically normal with mean zero and covariance matrix $K_R = I_R^{-1}$ where the element of I_R corresponding to $R_{jk}(v)$, $R_{\ell m}(u)$ is given by

$$\int_{-\pi}^{\pi} f_{km}^{-1}(\omega)\, f_{\ell j}^{-1}(\omega)\, e^{-i(v-u)\omega} d\omega$$

Further, K_R is consistently estimated by

$$\hat{K}_R = \left[\sum_{t=1}^{N} X^*(\omega_t)\ \hat{V}^{-1}(\omega_t)\ X(\omega_t) \right]^{-1}$$

3. CONCLUSIONS

In many applications, estimates of the spectral density are of interest, thus making available the quantities required by the proposed algorithm. Further the closeness to singularity of $\hat{f}$ provides an indication of whether the series generating the data is poorly behaved, i.e. whether the zeros of

$$|H(z)| = \left| \sum_{k=0}^{q} B(k) z^k \right|$$

are close to the unit circle. If this is the case, significantly improved estimators can be obtained by using the $\hat{R}$ as initial estimators in a direct maximization of the exact likelihood function.

Using the complex sweep operator (Parzen (1967)) one can find $\hat{f}^{-1}(\omega_j)$ by successively finding the inverse of various partitions of $\hat{f}(\omega_j)$, thus simultaneously finding estimators for various subsets of the components of $\underset{\sim}{Y}(t)$.

Finally, the algorithm of Wilson (1972) can be used to find estimators $\hat{B}(1), \ldots, \hat{B}(q)$, and $\hat{\Sigma}$ from $\hat{R}$.

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